Cantilevered Unsymmetric Fiber Composite Laminated Plates

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Theme

CANTILEVERED flat plate fiber composite laminates are convenient simulations for structural components such as: wings, horizontal and vertical stabilizers for airplanes, helicopter and wind mill blades, and compressor fan blades for aircraft engines. The traditional and current practice is to make and analyze the aforementioned components from laminate configurations which are balanced (no in-plane shear-stretch coupling) and symmetric (with respect to bending. However, the theoretical investigation of flat laminates with coupling responses is important because: 1) these responses may be used to design components with inherent self-damping mechanisms, and 2) warped structures from inadvertant misalignment errors during fabrication need by analyzed. Thus this investigation focuses on the prediction of the displacements and vibrations of cantilever flat laminates with various types of coupling responses. The predictions were obtained via the use of a special six node flat plate isoparametric finite element with five degrees-of-freedom (DOF) per node (3 translations and 2 rotations, and accounting for transverse shear, for anisotropic, and for inhomogeneous material. The applicability of the reduced-bending-stiffness concept, in conjunction with the finite element, in order to approximate the coupling response of cantilever composite plates is also investigated.

Contents

Ten different laminate configurations were selected for investigation. The ply orientations for these laminate configurations are shown schematically in Fig. 1; 12 plies of equal thickness were chosen for these laminates. Four cases are symmetric with respect to bending (cases I to III, and V); the other 6 cases are nonsymmetric and exhibit coupling. Note that case I has all 12 plies at 0°, and case V has 6 plies at 0°, and 6 plies at 90°. The laminate geometry is: 2-in. long, 1-in. wide, and 0.1-in. thick.

The displacement and vibration responses were investigated using different sets of displacement combinations in the analysis. These included: all 5 displacement DOF per node, 3 DOF per node (bending displacements only), 3 DOF per node (bending displacements only) with reduced bending stiffness. The results obtained include nodal displacements due to a concentrated load at the leading edge tip, and the first 6 natural frequencies with their corresponding mode shapes.

Tip deflection from the leading edge (LE) to the trailing edge (TE) at the quarter stations for the cantilever without bend-stretch coupling (only bending variables) are given in Table 1a. The finite element model for this case consisted of 16 elements with 3 displacement DOF (bending only) per node resulting in 120 free variables. The applied load for all the displacement results was a 15-lb concentrated load at the tip of the leading edge. The aluminum case is included to illustrate the corresponding behavior of a typical isotropic material. In Table 1b displacement results are shown using 5 DOF per node (200 variables total) for cases with coupling only. The effect of bend-stretch coupling (nonsymmetry) is evident when the displacements for cases IV, VI to X given in Table 1b are compared with the corresponding results given in Table 1a. Case I is the orthotropic case where all 12 plies are in the longitudinal direction. This is the stiffest case and exhibits a slight downward displacement at the trailing edge. Case X has very slight material bend-stretch coupling but it is noted that the trailing edge has a displacement larger than that obtained at the leading edge where the load is applied. With the ply orientation of cases IV and X, deflections are obtained which are very large compared to the orthotropic case. In Table 1c displacement results are shown for the material coupling cases using the reduced bending stiffness. Only the bending variables are used with this case. Thus, the total number of variables decreases from 200 to 120. As can be seen the results obtained are comparable to those given in Table 1b.

The previous discussion leads to the following important observation. Since the deformed shape for unsymmetric laminates is calculable, laminate configurations may be selected with predetermined twist (untwist) for anticipated mem-

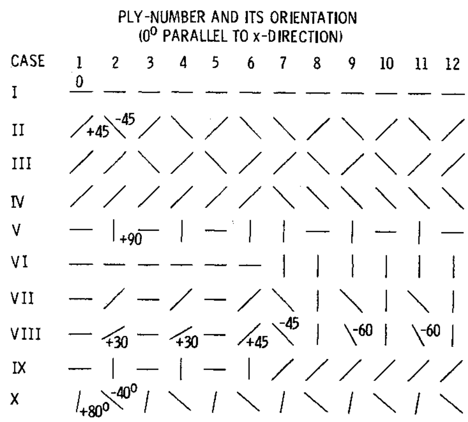

Fig. 1 Ply orientation schematic.

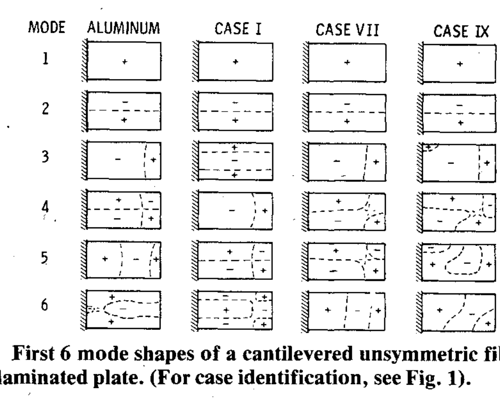

Fig. 2 First 6 mode shapes of a cantilevered unsymmetric fiber composite laminated plate. (For case identification, see Fig. 1).

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Index categories: Structural Composite Materials (including Coatings); Structural Dynamic Analysis; Structural Static Analysis.

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Table 1 Tip deflection of a cantilevered unsymmetric fiber composite laminated plate (in.) (For ply orientation, see Fig. 1)

a) Without coupling (3DOF, 16 element model)

Location	Case										
	Aluminum	I	II	III	IV	V	VI	VII	VIII	IX	X
LE	0.0510	0.0291	0.0878	0.1020	0.0838	0.0355	0.0405	0.0325	0.0366	0.0452	0.108
¼	0.0483	0.0184	0.0869	0.0986	0.0837	0.0262	0.0306	0.0293	0.0319	0.0385	0.116
½	0.0453	0.0099	0.0851	0.0950	0.0826	0.0172	0.0212	0.0264	0.0275	0.0326	0.124
¾	0.0432	0.0038	0.0825	0.0908	0.0807	0.0088	0.0123	0.0240	0.0234	0.0275	0.132
TE	0.0408	−0.0014	0.0788	0.0856	0.0777	0.0007	0.0036	0.0217	0.0194	0.0229	0.140

b) With coupling (5 DOF, 16 element model)

Location	Case					
	IV	VI	VII	VIII	IX	X
LE	0.150	0.0887	0.0562	0.0771	0.117	0.108
¼	0.146	0.0762	0.0533	0.0724	0.101	0.117
½	0.143	0.0644	0.0511	0.0685	0.087	0.124
¾	0.139	0.0536	0.0497	0.0656	0.075	0.133
TE	0.134	0.0434	0.0487	0.0361	0.063	0.141

c) With reduced stiffness (3 DOF, 16 element model)

Location	Case					
	IV	VI	VII	VIII	IX	X
LE	0.146	0.091	0.060	0.079	0.109	0.109
¼	0.143	0.078	0.057	0.074	0.095	0.116
½	0.139	0.066	0.055	0.070	0.082	0.124
¾	0.136	0.055	0.053	0.067	0.071	0.132
TE	0.131	0.045	0.052	0.065	0.061	0.140

Table 2 Natural frequencies of a cantilevered unsymmetric fiber composite laminated plate (cycles/sec) (For ply orientation see Fig. 1)

a) Without coupling (3 DOF, 4-element model)

Frequency order	Frequency magnitude for case									
	I	II	III	IV	V	VI	VII	VIII	IX	X
1	2092	916	840	936	1699	1537	1398	1411	1310	613
2	2786	4499	4025	4716	2565	2470	4160	3832	4069	3406
3	6327	5329	5096	5300	8865	8259	7803	7636	7136	5125
4	10109	11073	10186	11364	11879	11320	13202	13202	13034	9780
5	13053	15077	15203	14754	16575	16203	16008	15605	15626	11019
6	14738	18740	17084	19058	19154	20398	20349	20371	18265	21141

b) With coupling (5 DOF, 4-element model)

Frequency order	Frequency magnitude for case					
	IV	VI	VII	VIII	IX	X
1	647	892	974	867	813	611
2	3152	2078	3445	3172	2867	3390
3	3803	5322	6326	5551	4671	5101
4	7207	8007	10438	9716	9601	6344
5	8048	8359	11285	11296	9741	9727
6	10811	12025	14374	13311	12124	10994

brane and bending loads. This can be used to offset increasing angle of attack in airfoil designs and thereby provide structural damping to minimize or avoid flutter. Also the predetermined deformation can be used to contour the laminate fabrication mold.

The first 6 natural frequencies of the cantilever plate for the ten different ply orientations are summarized in Table 2a. These results are obtained using only bending variables, with the 4-element model and 3 DOF per node. In Table 2b, results are given for a 4 element model with 5 DOF per node. Note that the natural frequency of the orthotropic case is the highest for first frequency (first bending) but is low compared to the other cases for the second frequency (first torsion). For the cases with material coupling (Table 2b) all the modes are lower than those without coupling (Table 2a). The maximum difference that is obtained for the first frequency (using the case with coupling as a baseline) is 72% in case VI, while there is a 91% difference in the sixth frequency for case X.

A pictorial representation of the first six mode shapes is shown in Fig. 2. The mode shapes are normalized with respect to the leading edge tip displacement. Note that for case I, the third mode is a transverse bending mode which does not appear in the isotropic material (aluminum) until mode 6. Also note that the third bending mode of the isotropic material does not appear in the first six modes of case I. Case VII was chosen because it has significant material coupling while case IX has strong bending stiffness and complete coupling characteristics. Compared to the isotropic material, there is a slight shift in the nodal line for the second bending mode; the third bending mode appears as the sixth mode for cases VII and IX.

The important observations from the previous discussion are: 1) If coupling is not accounted, the cantilever predicted displacements are overly underestimated and the corresponding natural frequencies are overly overestimated. 2) Finite element codes which do not account for bend-stretch coupling can be used to predict displacements and vibration frequencies of unsymmetric laminates using the reduced-bending stiffness.